**7x +11y=100 Investigation**

**Case #1: Positive Integer values for x and y.**

**Obtaining the data:**

To obtain the possible positive integer values for x and y in a quick easy way, I will only work with y, meaning I will only work with multiples of 11 which are less than 100. I cannot work with multiples of 11 which are greater than 100 because I am only allowed to work with positive integers in case #1 (100-11y = 7x). Working with multiples of 11 will make the process of obtaining the data faster because I won’t have to work with two different sets of multiples (for 7 and 11) but, instead I will only focus in one. Working with multiples of 11, instead of multiples of 7, will allow me to obtain the possible values that satisfy the equation faster because there are more multiples of 7 which are less than 100 **(14 multiples),** than multiples of 11 which are less than 100 **(9 multiples).**

First, I will list all the **multiples of 11** which are **less than 100** **[11y < 100]:**  
  
Then, I will **subtract** each of the multiples I have listed from **100 [7x = 100-11y]:**  
Finally, I will **divide** the results of the subtractions above by **7** **[x = (100-11y)/7]:**  
**(Answers rounded to 3 significant figures.)**

|  |  |  |
| --- | --- | --- |
| **11y < 100** | **7x = 100-11y** | **x = (100-11y)/7** |
| 11\*1 = 11  11\*2 = 22 11\*3 = 33 **11\*4 = 44**  11\*5 = 55 11\*6 = 66 11\*7 = 77 11\*8 = 88 11\*9 = 99 | 100-11 = 89 100-22 = 78 100-33 = 67 **100-44 = 56** 100-55 = 45 100-66 = 34 100-77 = 23 100-88 = 12 100-99 = 1 | 89/7 = 12.7 78/7 = 11.1 67/7 = 9.57 **56/7 = 8** 45/7 = 6.43 34/7 = 4.86 23/7 = 3.29 12/7 = 1.71 1/7 = 0.14 |

The problem asks for the positive **INTEGER** values that satisfy the equation. In my answers above (third column), there is only one positive integer, number **8**. This demonstrates that there is only one possible value for x and one possible value for y which satisfies **case 1**.  
**X = 8**  and **Y=4.**

I can prove this by substituting them into the equation: **7x + 11y = 100**.  
7(8) + 11(4) = 100  
56 + 44 = 100  
100 = 100

**Case #2: Negative Integer values for X and positive integer values for Y**

**Obtaining the data:**

Obtaining the data for **case #2** will be more difficult than for **case #1**. This is because there will be an infinite number of possible values for x and y. In this case, I will have to find a pattern between the different values for x and different values for y, which satisfy the equation (7x+11y=100). After finding a pattern, I will write nth term formulas which describe how to get from a value of x and y to the next values. The method of only working with multiples of one number will also help us to obtain the data in this case. I will resume the list of multiples of 11 which I used in **case #1**. I will write down an adequate number of multiples of 11 (17 multiples) which will help me to notice a pattern between the values of x and y.

First, I will find **multiples of 11** which are **greater than 100 [11y > 100].**Then, I will **subtract** each of the multiples I have listed from **100** **[7x = 100-11y].**I will **divide** the results of the subtractions above by **7** **[x = (100-11y)/7].**  
**(Answers rounded to 3 significant figures.)**

|  |  |  |
| --- | --- | --- |
| **11y > 100** | **7x = 100-11y** | **[x = (100-11y)/7]** |
| 110 **121** 132 143 154 165 176 187 **198** 209 220 231 242 253 264  **275** 286 | 100-110 = -10 **100-121 = -21** 100-132 = -32 100-143 = -43 100-154 = -54 100-165 = -65 100-176 = -76 100-187 = -87 **100-198 = -98** 100-209 = -109 100-220 = -120 100-231 = -131 100-242 = -142 100-253 = -153 100-264 = -164 **100-275 = -175** 100-286 = -186 | -10/7 = -1.43 -**21/7 = -3** -32/7 = -4.57 -43/7 = -6.14 -54/7 = -7.71 -65/7 = -9.29 -76/7 = -10.9 -87/7 = -12.4 **-98/7 = -14** -109/7 = -15.6 -120/7 = -17.1 -131/7 = -18.7 -142/7 = -20.3 -153/7 = -21.9 -164/7 = -23.4 **-175/7 = -25** -186/7 = -26.6 |

The only **INTEGERS** in the answers above (third column) are **-3**, **-14** and **-25**.

The answer **-3** came from the equation 11\***11**. This means that the value of y that is the pair of -3 is 11. (x = -3, y=11)  
The answer **-14** came from the equation 11\***18**. This means that the value of y that is the pair of -14 is 18. (x = -14, y = 18)  
The answer **-25** came from the equation 11\***25.** This means that the value of y that is the pair of -25 is 25. (x = -25, y = 25)

Now that we have three values for x and three values for y, we have to find a pattern. To achieve this, I listed all the values of x in one column and the values of y in another.

**x = -3 y = 11** I looked at the values and calculated the difference between the values  **x = -14 y = 18** for x and for y (-3 - -14 = **11**, -14 - -25 = **11**. 18-11 = **7**, 25-18 = **7.)**   
**x = -25 y = 25** and I noticed a pattern.

The values of **x decrease by 11** as the values of **y increase by 7**.

After realizing this, I made nth term formulas that describe the pattern in the values of **x** and for the values of **y**, using **-3** and **11** as my first term.

**x = -11n + 8.**   
The values decrease by 11 each time. This means that the nth term is multiplied by -11. However, an 8 is added each term. **(How I figured it out: -11(1)+a | -11+a = -3 | a = -3+11 | a=8)** I tried the equation with each value of **x** I obtained (Ex: **-25 = -11(n)+8** | -25-8 =-11(n) | -33/-11 = n | **3 = n**) and it worked for each one I tried.

**y = 7n + 4.**   
The values increase by 7 each time. This means that the nth term is multiplied by 7. A 4 is added each term. **(How I figured it out: 7(1)+4 = b | 7+b = 11 | b = 11-7 | b = 4)** I tried the equation with each value of y I obtained (Ex: **25 = 7(n)+4** | 25-4 = 7(n) | 21/7 = n | **3 = n** ) and it worked for each one I tried.

Finally, I tried to **substitute** each of the equation I got above into the equation **7x+11y = 100**, when **n=100** to see if it worked.

7(-11(100)+8) + 11(7(100)+4) = 100  
7(-1100+8) + 11(700+4) = 100  
7(-1092) + 11(704) = 100  
-7644 + 7744 = 100  
100 = 100

*My formulas work.*

**Case #3: Positive Integer values for X and negative integer values for Y**

**Obtaining the data:**

For **case #3,** there will also be an infinite number of possible values for x and y which satisfy the equation **7x+11y = 100**. I will have to find a pattern between the values of x and y, and then make nth term formulas which describe how to get from a value of x and y to the next value for this case too. In this case, I will have to work with **negative multiples of 11** to find the possible positive values for x, which satisfy the equation. Working with multiples of 11 is easier (as I stated in **case #1**) because it is a bigger number than 7, meaning I will work with less multiples. I will list an adequate number of negative multiples of 11 (17 multiples) which will help me notice a pattern.

First, I will find **negative multiples of 11 [11y < 0].**Then, I will **subtract** each of the multiples I have listed from **100** **[7x = 100-11y].**I will **divide** the results of the subtractions above by **7** **[x = (100-11y)/7].**  
**(Answers rounded to 3 significant figures.)**

|  |  |  |
| --- | --- | --- |
| **11y < 0** | **7x = 100-11y** | **x = (100-11y)/7** |
| -11 -22 **-33** -44 -55 -66 -77 -88 -99 **-110** -121 -132 -143 -154 -165  -176  **-187** | 100--11 = 111 100--22 = 122 **100--33 = 133** 100--44 = 144 100--55 = 155 100--66 = 166 100--77 = 177 100--88 = 188 100--99 = 199 **100--110 = 210** 100--121 = 221 100--132 = 232 100--143 = 243 100--154 = 254 100--165 = 265 100--176 = 276 **100--187 = 287** | 111/7 = 15.9 122/7 = 17.4 **133/7 = 19** 144/7 = 20.6 155/7 = 22.1 166/7 = 23.7 177/7 = 25.3 188/7 = 26.9 199/7 = 28.4 **210/7 = 30** 221/7 = 31.6 232/7 = 33.1 243/7 = 34.7 254/7 = 36.3 265/7 = 37.9 276/7 = 39.4 **287/7 = 41** |

The only **INTEGERS** in the answers above (third column) are **19**, **30** and **41**.

The answer **19** for x came from the equation 11\***-3**. This means that the value of y that is the pair of 19 is -3. (x = 19, y=-3)  
The answer **30** came from the equation 11\***-10**. This means that the value of y that is the pair of 30 is -10. (x = 30, y = -10)  
The answer **41** came from the equation 11\***-17.** This means that the value of y that is the pair of 41 is -17. (x = 41, y = -17)

Now I have 3 positive values of x and 3 negative values of y. I have to find the pattern between the values. I listed the pairs from the biggest to smallest value of x (smallest to biggest value of y) like I did on **case #2.**

**x = 41 y = -17** I calculated the difference between value to value for x and for y to find  **x = 30 y = -10** a pattern between the values. (41-30 = **11**, 30-19 = **11**. -3--10 = **7**,   
**x = 19 y = -3**  -10--17 = **7.)** The pattern is similar to the one on **case #2**.

The values of **x increase by 11** as the values of **y decrease by 7**.

I made nth term formulas which describe the pattern in which you get from value to value of x and y. I used the closest values to 0, x= 19 and y= -3, as the first terms.

**x = 11n + 8.**   
(It is almost the same to the formula in **case #2.** The only difference is that n is multiplied by a negative 11 in **case #2,** while in **case #3,** it is multiplied by a **positive** 11.)   
I noticed that the values increase by 11 each time. This indicates that the nth term is multiplied by 11. An 8 is added each term. **(How I figured it out: 11(1)+c=19 | 11+c = 19 | c=19-11 | c=8)** I tried the equation with each value of **x** I obtained (Ex: **41 = 11(n)+8** | 41-8 =11(n) | 33/11 = n | **3 = n**) and it worked for each one I tried.   
  
**y = -7n + 4.**(This equation is also similar to the one in **case #2.** The only difference is that n is multiplied by a negative 7 in this case (in **case #2** it was multiplied by a positive 7).)  
The values decrease by 7 each time, it indicates that the nth term is multiplied by a negative 7. A 4 is added each term. **(How I figured it out: -7(1)+d=-3 | -7+d = -3 | d=-3+7 | d=4)** I tried the equation with each value of y I obtained (Ex: **-17 = -7(n)+4** | -17-4 = -7(n) | -21/-7 = n | **3 =n** ) and it worked for each one I tried.

To prove that my formulas work, I **substituted** x and y with the formulas I found on the equation **7x +11y = 100,** when **n = 200**.  
  
7(11(200)+8) + 11(-7(200)+4) = 100  
7(2200+8) + 11(-1400+4) = 100  
7(2208) + 11(-1396) = 100  
15456 + -15356 = 100  
100 = 100  
 *My formulas work.*

To conclude, the final fomulas I got were:  
**1. When x is a negative integer and y is a positive integer:**

**x = -11n+8  
y = 7n+4  
  
2. When x is a positive integer and y is a negative integer:**

**x = 11n+8  
y = -7n+4**

I also tried my equations with a random number like 123456 for n to prove they work.

**When x is a negative integer and y is a positive integer:**  
7(-11(123456)+8) + 11(7(123456)+4) = 100  
7(-1358016+8) + 11(864192+4) = 100  
7(-1358008) + 11(864196) = 100  
-9506056 + 9506156 = 100  
100 = 100

**When x is a positive integer and y is a negative integer:**7(11(123456)+8) + 11(-7(123456)+4) = 100  
7(1358016+8) + 11(-864192+4) = 100  
7(1358024) + 11(-864188) = 100  
9506168 + -9506068 = 100  
100 = 100